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The Effect of Memory on Clustering in Spontaneous Aggregation

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ABSTRACT

The effect of short-term and long-term memory on spontaneous aggregation of organisms is studied using an agent-based stochastic model, where each individual modulates its random movement based on the perceived local density of its neighbours. Memory is introduced via a chain of-K internal variables, allowing agents to retain past environmental information. The number of internal variables, K, controls the memory length.

The macroscopic Fokker-Planck equation is formally derived in the large-particle limit. To explain the formation and shape of particle clusters, the steady states of the Fokker-Planck equation are characterized, and systematic stochastic simulations of the individual-based model performed.

The short-term memory promotes cluster coarsening, while long-term memory disrupts aggregation, increasing the number of outliefs and instances with no clustering. Statistical analysis shows that memory inhibits the particles' responsivity to environmental cues, specifically the perceived density of their neighbours, explaining the reduced clustering tendency at higher values of the memory length, K. This study therefore provides an insight into how memory influences emergent spatial patterns in self-organizing systems of organisms.

Keywords. Memory effects, Fokker-Planck equation, Collective behaviour.

Introduction

Models with memory and delays are ubiquitous in explaining collective behaviour of organisms, ranging in scale from bacterial and amoeboid chemotaxis to behaviour of social insects and flocks of birds. In the case of bacteria, the memory is incorporated in purely biochemical terms, in the form of the signal transduction network capable of excitation and adaptation dy-namics. This enables a cell to 'memorize' past environmental signals and compare to their current state to inform decision making.

In individual-based models, such memory effects can be accounted for by ordinary differential equation (ODE) or stochastic differential equation (SDE) models describing internal variables of each individual. A typical example of such internal variables are concentrations of key intracellular

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biochemical species.[i] At the collective (macroscopic) F level, populations of bacteria or other cells can be described by partial differential equations (PDEs) describing the density of cells and key environmental signals, where memory can also be interpreted as delays in the processing of some signals.

Incorporating such delays into mathematical models of collective phenomena can provide more accurate explanations of behavioural properties of groups of animals[ii] or robots. Moreover, Non-Markovian characteristics resulting from ordering of interactions in temporal complex networks were identified as an important mech anism that alters causality and affects dynamical processes in social, technological and biological systems. Considering biological agents (cells or animals), a trade off between having no memory at all and remembering too much (including very distant and irrelevant past states) has been achieved by evolution. For example, bacterial biochemical memory of past environmental signals evolved to last for the duration of a few seconds.

In this paper we focus on a model of spontaneous aggregation of animals or cells resulting from random diffusive motion of individuals. The individuals respond to the local population density observed in their neighbourhood by increasing or decreasing the amplitude of their random motion. This kind of behaviour has been observed in insects, for instance, the pre-social German cockroach (B. germatica), which are known to be attracted to dark, warm and humid places. However, it has been shown that cockroach larvae also aggregate spontaneously, i.e., in the absence of any environmental template or heterogeneity. Moreover, this type of dynamies can also be used to describe formation of P Granules in C. elegans embryos resulting in spontaneous protein aggregation.

To incorporate memory into the first-order model of spontaneous aggregation, we introduce a set of $K \geq 1$ internal variables of each agent. Each internal variable describes a 'layer' of memory. which is subject to two effects: (i) production (or excitation) from the internal variable of the next higher order, and (ii) spontaneous decay with a constant rate. The internal variable of the highest (Le., K-th) order is then subject to random excitation, with amplitude modulated by a nonlinear function of the number density of agents observed in the physical neighbourhood. We note that in the first-order model introduced in, the positions of the agents in the physical space were directly subjected to random excitation (Brownian motion). In particular, our investigation extends their model by introducing a chain of K internal variables that allow the agents to 'remember' the densities they encountered in the past.

Our results show that the introduction of memory with a few layers, K, leads to the formation of a smaller number of larger clusters of agents. This trend is observed until K3 or K=1. depending on the spatial dimension of the studied system, as shown in Sections 5 and 6. When the number of memory layers K is increased further, i.e., as the memory becomes 'longer', its effect starts to be disruptive. This is manifested by the increasing proportion of 'outliers', which are the agents that are not part of any cluster. We therefore conclude that short-term or medium-term memory has a coarsening effect on spontaneous aggregation, while long-term memory disrupts it.[iii]

The paper is organized as follows. In Section 2 we describe the (first-order) spontaneous ag gregation model and summarize its main properties studied in the literature [6]. In Section 3, we introduce memory into this model and infer the main mathematical properties of the model with memory. In Section 4, we then derive the formal macroscopic limit of the system as the number of agents tends to infinity, obtaining the corresponding Fokker-Planck equation. We characterize its steady states to gain an insight into the patterns (clusters) formed by the system. In Section 3. we report the results of the stochastic simulations of the individual-based model in the spatially one-dimensional setting, while the two-dimensional results are presented in Section 6.[iv] Here we also provide a statistical evidence that the long-term memory inhibits the particles' responsivity to environmental stimuli. We conclude with the discussion in Section 7.

Aggregation model and spatial dimension

The individual-based stochastic model introduced in reference (6) under the name 'direct aggregation model consists of a group of N≥ 2 biological agents (cells or animals), characterized by their positions x_{i}(t) \in \mathbb{R} ^ d with spatial dimension d \in \{1, 2, 3\} and i \in [N] where we have denoted the set of indices by [N]:=\ 1, 2 ,...,N\ . Every individual senses the average density of its close neighbours, given by

$$\vartheta_i(t) = \frac{1}{N} \sum_{j\in[N]} W(\mathbf{x}_i(t) - \mathbf{x}_j(t)), \qquad \text{for} \quad i \in [N],$$

where W(x) = w(|x|) with the weight function w : mathbb R ^ + mathbb R ^ + assumed to be bounded, nonnegative, nonincreasing and integrable on \mathbb{R} ^ d Without loss of generality we impose the normalization.

$$\int_{\mathbb{R}^d} W(\mathbf{x})\,\mathrm{d}\mathbf{x} = 1\,.$$

A generic example of w is the (properly normalized) characteristic function of the interval (0. R), corresponding to the sampling radius R > 0 The average density vartheta_{i} is then simply the fraction of individuals located within the distance R from the i-th individual. The individual positions are subject to a random walk with modulated amplitude, described by the system of coupled SDES.

$$dx_i(t) = G(\vartheta_i)\, dB_i^t, \qquad \text{for} \quad i \in [N],$$

Impact of the Clustering Memory

As the dimensionality of the Fokker-Planck equation (4.1) becomes prohibitive for numerical simulations even with moderate values of K, we use stochastic simulations of the individual-based model given by equations (2.1) and (3.1) to systematically investigate the impact of the number of memory layers K >= 1 on the clustering properties of the spontaneous aggregation model. We use epsilon_{k} and alpha_{k} , for k \in [K] given by equations (3.3) and (3.6), respectively, where we choose alpha = 1 in equation (3.6). The distance between agents is calculated over the torus, i.e., taking into account the periodic boundary conditions on 2. We simulate N = 400 agents moving in the domain (4.4) for d = 1 The response function G(s) and the interaction kernel W(x) are given by (4.9) with the sampling radius R = 1/40 = 0.025.[v]

We initialize the simulation by randomly generated agent positions x_{i} \in Omega for i \in [N] using uniformly distributed initial positions in Ω. All internal variables are initialized as zeros, ie., y_{i} ^ k * (0) = 0 for all i \in [N] and k \in [K] We discretize equations (3.1) using the Euler-Maruyama with timestep Delta*t = 10 ^ - 3 We calculate 100 stochastic realizations of the individual-based model in the time interval [0, 10 ^ 3] i.e., we calculate the time evolution over 10 ^ 6 timesteps for each realization. We record the positions at the final time t = 10 ^ 3 and use these for evaluating clustering properties.

Discussion

In this paper, we have shown that short-term memory enhances and long-term memory inhibits spontaneous particle aggregation. Our investigation has been based on the (first-order) spontaneous aggregation model without memory which has been previously investigated in the literature [6]. Its main properties are summarized in Section 2. The memory has been added into this model in Section 3 by introducing a chain of K internal variables that allow the agents to 'remember' the densities they encountered in the past. If K = 1, then our individual-based model is equivalent to what is called 'the second-order model' in reference, where the internal variable represents the agent's velocity.

Linear and Nonlinear Function

It is considering K > 1, the model introduces additional layers of memory described by K internal variables. Additional internal variables can be used to better fit the properties of relatively complex (high-dimensional) models of interacting particle systems, while keeping the number of degrees of the system (internal variables) relatively small. For simplicity, our transfer of information between layers of memory is linear, but nonlinear functions can also be introduced to better fit the properties of some systems.

References:

1. Mathematical Institute, University of Oxford, Radcliffe Observatory Quarter, Woodstock Road, Oxford, 2021 p. 56
2. Mathematical and Computer Sciences and Engineering Division, King Abdullah University of Science and Tech-nology, 2019 p. 39
3. X. Mao. Stochastic Differential Equations and Applications. Elsevier, Amsterdam, Netherlands, 2007
4. K. Gurney. An introduction to neural networks. CRC press, 2018.
5. D. Sumpter. Collective Animal Behavior. Princeton University Press, 2010.