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Fuzzy Computational Analysis of Fuzzy hybrid structure

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ABSTRACT

In this paper, we explore the application of fuzzy topological indices to the fuzzy tadpole graph , a hybrid structure that combines the cyclic and path-like components within a fuzzy framework. We compute and analyze several key fuzzy indices, including the fuzzy Zagreb indices, fuzzy Randić index, fuzzy Harmonic index, fuzzy Sombor index, fuzzy Misbalance Prodeg index, and fuzzy Nirmala index, to gain a comprehensive understanding of the graph structural and topological properties.

Our results reveal how fuzzy indices provide deeper insight into the graph connectivity and stability, especially in scenarios where traditional (crisp) indices may be inadequate. These findings have potential applications in domains such as complex network analysis, molecular chemistry, and systems with inherent uncertainty.

Keywords:- Framework, Comprehensive, Properties, Especially

Graph theory is the basic concept in the study of complex systems, providing a mathematical framework for simulating links and interactions in disciplines such as computer science, chemistry, biology, and social science. Topological indices (TIs) are numerical variables linked with a graph's structure that capture its fundamental attributes. These indices are rapidly employed in fields ranging from network analysis to molecular property prediction in chemical informatics.

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Traditional TIs are typically used for crisp networks in which edges are either present or absent, leaving no space for ambiguity. However, many real-world systems are characterized by uncertainty, imprecision, and ambiguity, which crisp graph models cannot portray. To address this restriction, fuzzy graph theory extends traditional graph notions by allowing both edges and vertices to have varying degrees of membership, reflecting the structure's inherent ambiguity. Fuzzy TIs are a natural extension of classical indices in this context, allowing for a more refined analysis of graph topologies with connections that are not strictly binary but vary in strength. These indices are especially useful when the relationship between entities is unclear or when it's necessary to account for ambiguity in interaction strengths.

The purpose of this work was to look into fuzzy TIs for tadpole graphs. The tadpole graph is a form of graph in graph theory that consists of a single cycle (the "body" of the tadpole) and a single path (the "tail" of the tadpole) connected to the cycle. Our study seeks to assess the behavior of fuzzy TIs when applied to these graphs, specifically how these indices represent the structural nuances given by fuzziness. We wish to better grasp the benefits of fuzziness in graph analysis by comparing indices in both crisp and fuzzy contexts. This paper adds to the increasing body of research in fuzzy graph theory and its application by providing fresh insight into the structural analysis of complex systems with uncertainty as a basic component.

A graph $\mathcal{G}$ is an ordered pair $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ of:

- $\mathcal{V}$, a set of vertices (or nodes),
- $\mathcal{E}$, a set of edges, The points or vertices are the basic units of the graph, while the edges are the connections between them.

According to crisp graph theory, the number of edges incident to a vertex u in a graph is its degree. The degree of a vertex u, denoted by deg(u), in a graph $\mathcal{G}$ is usually explained as

$$\mathcal{G} = (\mathcal{V}, \mathcal{E}),$$

$$\deg(u) = |\{e \in \mathscr{E} \mid u \in e\}|,$$

Graph theory is the study of graphs, which are mathematical structures that represent pairwise relationships between things. Leonhard Euler, who solved the famous Königsberg bridge problem in 1736, is credited with pioneering graph theory. Euler's work laid the groundwork for graph theory, formulating the concept of a graph as a collection of vertices connected by edges.[1] Fuzzy graph theory is an extension of classical graph theory that covers situations in which interactions between entities are ambiguous, imprecise, or unclear. In 1965, Lotfi A. Zadeh combined fuzzy set theory with graph theory to describe edges in a graph with varying degrees of membership instead of binary (0 or 1) values. This paradigm is particularly useful in areas with uncertain linkages, such as the social sciences, biology, and decision-making processes.

Fuzzy graphs were first introduced by Arthur Rosenfeld in 1975 Rosenfeld's seminal work used fuzzy sets to graphs, allowing each edge to have a degree of membership ranging from 0 to 1. Following Rosenfeld's work, the researchers investigated fuzzy graph operations, such as connectedness, matching, and color ability. Their findings are regarded as critical to the future development and use of fuzzy graph theory. Pal reviewed segmentation techniques that laid the groundwork for using fuzzy models in image analysis, supporting the role of fuzzy theory in handling uncertainty relevant to our fuzzy graph approach. Shukla et al. demonstrated fuzzy decision-making in supply chain, showing how fuzzy logic effectively manages complex, uncertain systems, which aligns with our use of fuzzy indices in structural graph analysis. Prabusankarlal et al. highlighted fuzzy-based methods in breast cancer diagnosis, illustrating how fuzzy models can aid in analyzing uncertain and imprecise data similar to fuzzy graph applications in network modeling . TIs, which are numerical values derived from the structure of a graph, are used to evaluate and classify various molecular structures and network features.

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These indices offer a significant improvement in graph theory, merging mathematical and chemical principles to provide useful modeling and analysis tools. TIs are numerical indices generated during the graphing process that are used to classify and investigate many features of molecular structures in chemistry and networks. These indices are a significant improvement in graph theory, combining notions from mathematics and chemistry to give a useful tool for modeling and analysis.

In 1947, a researcher named Harold Wiener created the concept of TIs. Wiener introduced the first topological index, Wiener index, when he was working on the boiling points of paraffin. To find the Wiener index, sum the distances between all of the graph's vertex pairs.[2] This index was initially used to investigate molecular characteristics and has since become one of the most often used indices in chemical indices in chemical graph theory. The Zagreb indices were among the earliest TIs developed to describe molecular structures in chemistry. The degrees of the graph's vertices are used to define the traditional first and second Zagreb indices, which are widely employed in quantitative structure-activity relationship (QSAR) research. Gutman and Trinajstic (1972)[3] were the first to apply these indices to fuzzy graphs, establishing topological index theory.

The authors evaluated the F-index, a topological index based on node degree, as well as the molecular descriptor. Mondal et al. studied various TIs based on neighborhood degree. TIs are widely employed in chemical and spectral graph theory, network theory, and molecular chemistry. Given their widespread use in crisp graphs, researchers are also investigating TIs in fuzzy graphs. In, Uzma et al. investigated and applied TIs in cybercrime. Degree-based TIs to analyze extremal values of benzenoid structures are studied in, whereas Kalathian discussed distance-based TIs in. In, a comprehensive overview of the Randić index is provided.

The application of standard TIs of fuzzy environments has opened up new possibilities for studying graphs with nonbinary links. Several TIs have been developed to capture various components of graph structure in the presence of uncertainty. Initially, experts focused on using standard TIs for fuzzy graphs, such as the Wiener, Randić, and Zagreb indices. As part of this technique, proper fuzzy representations for concepts like degree and distance were created. Aside from enhancing traditional indices, researchers developed entirely new fuzzy TIs to represent the distinct characteristics of fuzzy graphs. These indices typically aggregate the fuzzy memberships of vertices and edges to get a scalar value that represents a specific topological characteristic.

Kalathian et al. introduced degree-based TIs for fuzzy graphs in. Islam and Pal studied the first fuzzy Zagreb index and presented results for various fuzzy graphs, such as fuzzy path graphs, fuzzy cycle graphs, fuzzy star graphs, and fuzzy subgraphs. They also provide a technique for deciding which employee in a corporation is the most competent through multicriteria decision-making (MCDM), which takes advantage of fuzzy graphs' first Zagreb index. Poulik et al. developed the Randić index to analyze bipolar fuzzy graphs in. Akram et al. devised and applied the Randić index to bipolar fuzzy cycle graphs. Kalathian in used the fuzzy F index for fuzzy graphs to analyze railway crimes in India and compared it to three other TIs. In, researchers studied the fuzzy extension of TIs and found it useful for networks with unknown connections and fuzzy graphical structures. ZS Mufti formed the fuzzy Misbalance Prodeg index in and used it for MCDM. The fuzzy Sombor index has been examined in with applications in trade flows overseas.

Recent contributions have emphasized the importance of fuzzy graph indices in modeling uncertainty in networks and systems. Akram and Dudek presented fuzzy soft graphs for decision-making[4], while Rashmanlou and Pal introduced new indices within intuitionistic fuzzy graphs[5].

Similarly, Pal explored distance-based indices in q-rung orthopair fuzzy graphs, a generalization suitable for complex uncertainty modeling, and applications in molecular chemistry have also been analyzed using fuzzy TIs, and spectral approaches have recently been developed to enhance structural analysis. Mondal et al. applied TIs to model and evaluate chemical compounds used in the treatment of COVID-19, illustrating the practical importance of these indices in drug development and molecular chemistry.

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In this study, we provide a fuzzy alternative to standard TIs for fuzzy tadpole graphs. We examine fuzzy versions of well-known indices like the Zagreb index, Randić index, Harmonic index, Sombor index, Misbalance Prodeg index, and Nirmala index. These indices are customized to reflect the properties of graph types in a fuzzy context. This work attempts to better understand fuzzy tadpole graphs and how to use them by developing and assessing fuzzy TIs. The findings provided here shed fresh light on the structural analysis of fuzzy tadpole graphs, opening the door to future research and practical application in a wide range of fields.

Parameter	Description	Role in graph construction
$\mathcal{A}$	Number of vertices in cycle $C_{\mathcal{A}}$	Forms the cyclic part of the fuzzy tadpole graph.
$\mathcal{B}$	Number of vertices in path $P_{\mathcal{B}}$	Forms the linear path attached to the cycle.
$T_{\mathcal{A},\mathcal{B}}$	Fuzzy tadpole graph with total $\mathcal{A}+\mathcal{B}$ vertices	Combines $C_{\mathcal{A}}$ and $P_{\mathcal{B}}$ at one vertex.

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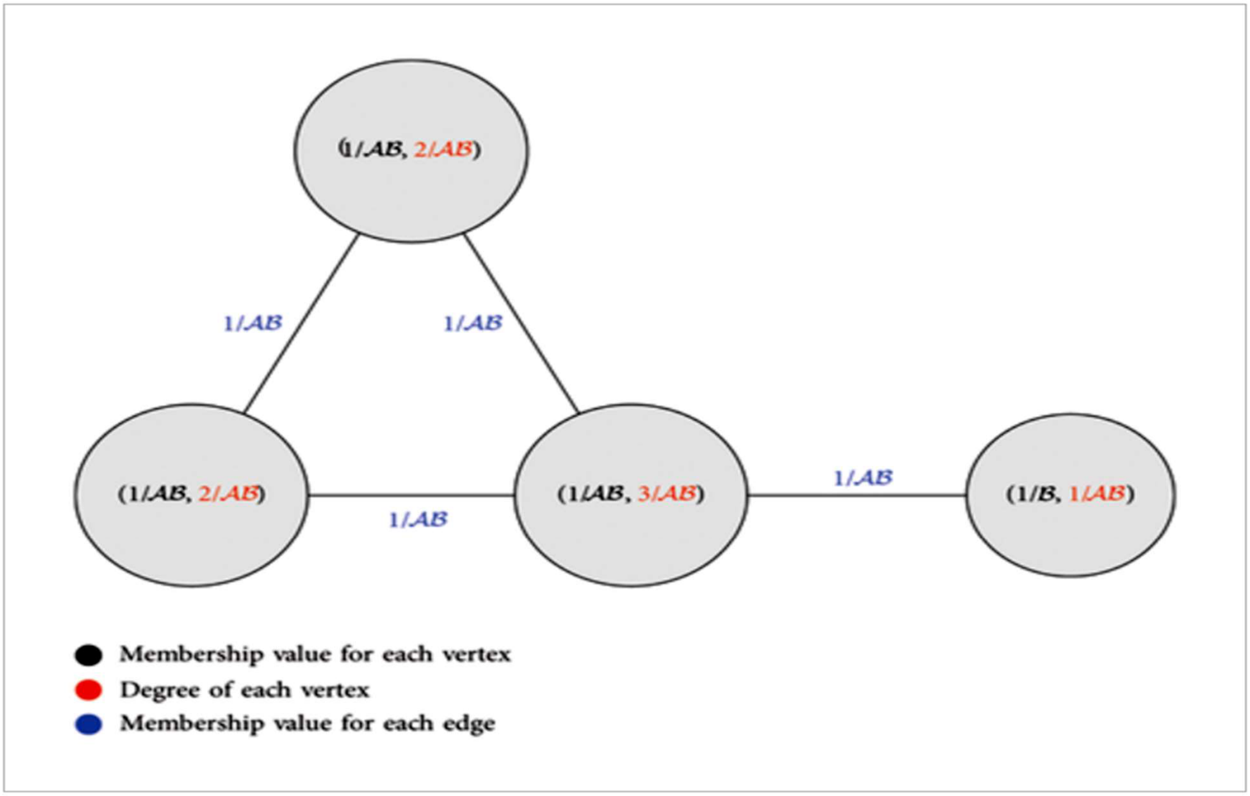

Conclusion

This study investigates the application of fuzzy indices on the fuzzy tadpole graph , providing new insights into its structural and topological properties. By analyzing indices such as the fuzzy Zagreb index, Randić index, Harmonic index, Sombor index, Misbalance Prodeg index, and Nirmala index, we have derived precise formulas that capture every aspect of the fuzzy tadpole graph in terms of its parameters and . We found and simplified explicit formulas for numerous fuzzy TIs of the fuzzy tadpole graph. These formulas provide a full understanding of how the graph's structure influences these indices, enabling further investigation and comparison with other graph types.

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Future research can explore the extension of these fuzzy indices to more complex graph structures, such as fuzzy barbell, helm, or triangular snake graphs. Additionally, incorporating advanced fuzzy logic frameworks like intuitionist, Pythagorean, or q-rung fuzzy models could yield even deeper insights. Application-based studies in fields such as molecular chemistry, social network analysis, and decision support systems may also benefit from these developments.

References

1. Fuzzy soft graphs for decision-making 1972
2. Pal introduced new indices within intuitionist fuzzy graphs
3. Randić index to bipolar fuzzy cycle graphs.
4. Zagreb indices were among the earliest
5. famous Königsberg bridge problem in 1736